

A Level Mathematics – Activity 1

Show your work!

Part 1

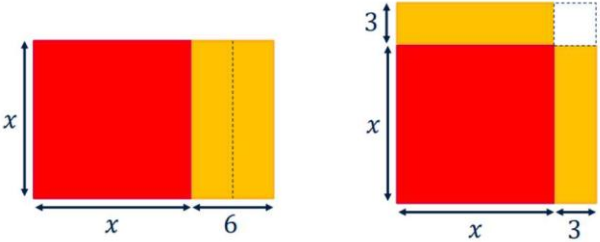
This is a set of algebra questions involving completing the square. I am sure you have heard ‘show your working out’ before, but communication and correct mathematical language/notation is extremely important at A Level.

Have a look at the diagrams which show completing the square pictorially.

These are different forms of the same algebraic expression

$$x^2 + 6x = x(x + 6) = (x + 3)^2 - 9$$

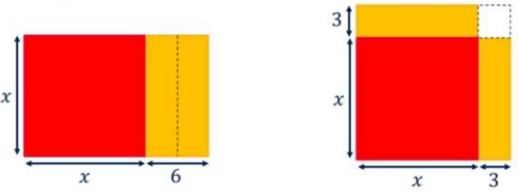
expanded form factorised form completed square form



Do the diagrams help you see why this is called **Completing the square?**

Think you've seen these diagrams before?

- They are very similar to the diagrams for the **Difference of Two Squares** – as seen previously in *Expanding Double Brackets*.



Can you see this is the difference of two squares?

$$(x + 3)^2 - 3^2$$

$$((x + 3) - 3)((x + 3) + 3)$$

Collect like terms within the brackets

$$= x(x + 6)$$

$$= x^2 + 6x$$

Solving a quadratic equation

Part 2 - watch this video:

<https://www.youtube.com/watch?v=voZOplT89wE&list=PL5pdglZEO3NhOCf8AVW78hQvJkmF8-am&index=5>

Part 3 - try these showing full mathematical solutions.

✔
Completing the square 1
✔

Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 4x$	5. $x^2 - 12x + 41$
2. $x^2 + 4x + 5$	6. $k^2 + 10k - 2$
3. $y^2 - 8y$	7. $y^2 + 3y + 1$
4. $y^2 - 8y + 7$	8. $p^2 - 2p + 1$

✔
Completing the square 2
✔

Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 10x$	5. $x^2 - 8x + 25$
2. $x^2 + 10x + 30$	6. $k^2 + 14k - 1$
3. $y^2 - 2y$	7. $y^2 + 5y + 6$
4. $y^2 - 2y + 3$	8. $t^2 + 6t + 9$

Part 4 - mark your work in a different colour with the following solutions

Complete the square 1 solutions

1. $x^2 + 4x$	→ $= (x + 2)^2 - 4$	5. $x^2 - 12x + 41$	→ $= (x - 6)^2 + 5$
2. $x^2 + 4x + 5$	→ $= (x + 2)^2 + 1$	6. $k^2 + 10k - 2$	→ $= (k + 5)^2 - 27$
3. $y^2 - 8y$	→ $= (y - 4)^2 - 16$	7. $y^2 + 3y + 1$	→ $= \left(y + \frac{3}{2}\right)^2 - \frac{5}{4}$
4. $y^2 - 8y + 7$	→ $= (y - 4)^2 - 9$	8. $p^2 - 2p + 1$	→ $= (p - 1)^2$

Complete the square 2 solutions

1. $x^2 + 10x$	→	$= (x + 5)^2 - 25$	5. $x^2 - 8x + 25$	→	$= (x - 4)^2 + 9$
2. $x^2 + 10x + 30$	→	$= (x + 5)^2 + 5$	6. $k^2 + 14k - 1$	→	$= (k + 7)^2 - 50$
3. $y^2 - 2y$	→	$= (y - 1)^2 - 1$	7. $y^2 + 5y + 6$	→	$= \left(y + \frac{5}{2}\right)^2 - \frac{1}{4}$
4. $y^2 - 2y + 3$	→	$= (y - 1)^2 + 2$	8. $t^2 + 6t + 9$	→	$= (t + 3)^2$

Part 5






Different Forms

It is important to be able to convert expressions between the different forms:

expanded form factorised form completed square form

In this problem there are 4 sets of three equivalent expressions, however, some expressions are missing. Match the sets and find the 3 missing expressions.

$a^2 - 2a - 8$		$a^2 - 8a + 15$
	$a^2 + 2a - 15$	$(a + 2)(a + 4)$
$(a + 1)^2 - 16$	$(a - 3)(a - 5)$	
$(a + 5)(a - 3)$	$(a - 1)^2 - 9$	$(a + 3)^2 - 1$



Extra Puzzle 1

What is the value of

$$\frac{\frac{(5^2 - 3^2)}{5 + 3} + \frac{(4^2 - 2^2)}{4 + 2} + \frac{(3^2 - 1^2)}{3 + 1}}{2} ?$$